



Using prompted self-explanations in first-year calculus

Kseniya Garaschuk and Costanza Piccolo
{kseniya, costanza}@math.ubc.ca



Background

A **worked example** is an example that involves a step-by-step solution to a problem. It can be presented in textual, graphical, video or face-to-face format (latter is sometimes called “expert modelling”).

A **self-explanation** or a **self-generated explanation** is an explanation of presented instruction that integrates the presented information with background knowledge and fills in tacit inferences.

A **prompted self-explanation** is a process by which, when given a piece of material to study, students are given external cues eliciting self-explaining.

Many studies show the effectiveness of both worked examples (Renal et al.), self-explanations (Chi et al.). However, there is a large amount of variance in the amount and quality of individually produced self-explanations. Prompting self-explanations is a more reliable technique and has been shown to work with:

- verbal prompts (Chi et al., 1994),
- computer-generated prompts (McNamara, 2004; Aleven and Koedinger, 2002; Hausmann and Chi, 2002),
- prompts embedded in the learning materials (Hausmann and VanLehn, 2007).

So far, none of the studies were applied to a service first-year calculus course.

Course

MATH 110 is a two-term, six-credit course in differential calculus that covers the same calculus content as the one-term courses, but with additional material designed to strengthen understanding of essential pre-calculus topics.

This course is meant for students who do not satisfy the prerequisites for one-term calculus courses and normally students with Pre-Calculus 12 grades higher than 85% are not permitted to take MATH 110.

Implementation

Why? Pilot study to test out materials and figure out controls for next year’s implementation.

Who? One section of MATH 110 with ~50 students.

What? Introduce prompted self-explanations as an active learning technique.

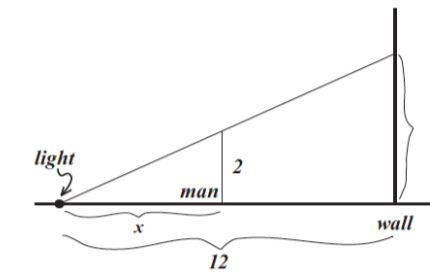
How? Provide worked examples and ask students to explain particular steps.

How often? One hour a week in class (worksheets) plus one question on every assignment.

Sample materials

Problem 1: A spotlight on the ground shines on a wall 12 meters away. If a two meter man walks from the spotlight to the wall at a speed of 1.6 meters per second, how fast is the length of his shadow on the wall decreasing when he is 4 meters from the wall?

Solution.
Let us first draw a diagram and label it.



Let x be the distance from the man to the light and let y be the height of his shadow. We are given that $\frac{dx}{dt} = 1.6$; we are asked to find $\frac{dy}{dt}$ when $x = 8$.

$$\frac{x}{2} = \frac{12}{y} \rightarrow \text{Why?}$$

$$xy = 24$$

$$\frac{d}{dt}(xy) = \frac{d}{dt}(24)$$

$$x \frac{dy}{dt} + y \frac{dx}{dt} = 0 \rightarrow \text{Why? What rule(s) did you use?}$$

$$\frac{dy}{dt} = -\frac{y}{x} \frac{dx}{dt}$$

When $x = 8$, we have $y = 3$. \rightarrow Why?

Therefore, $\frac{dy}{dt} = -\frac{3}{8}(1.6) = -0.6$ meters per second.
Does the answer make sense? Explain.

WRITTEN ASSIGNMENT 16

Hand in full solutions to the questions below. Make sure you justify all your work and include complete arguments and explanations. Your answers must be clear and neatly written, as well as legible (no long drawings or micro-handwriting please!). Your answers must be stapled, with your name and student number at the top of each page.

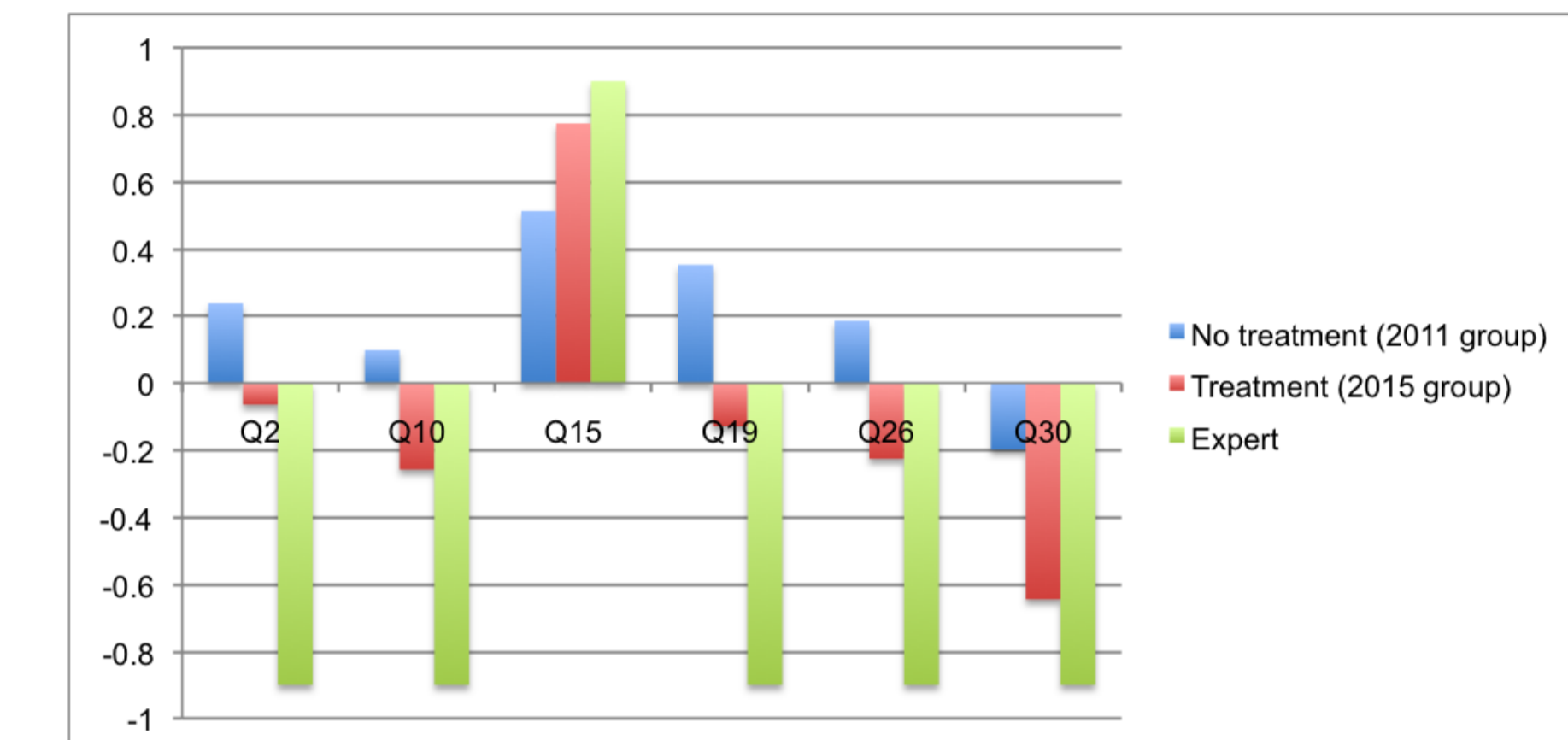
- This week we learned how to solve optimization problems. Explain in your own words what we mean by “optimization problems”, then provide an example of an optimization problem with a full solution. Your example must be an original problem (not from network, calculus textbooks, Google, etc.).
- Below is an optimization problem with solution. Your task is to:
 - draw a sketch representing the situation described in the problem (you should label quantities on your sketch consistently with the solution provided)
 - answer the handwritten questions.
 - show a different way to derive the equation for the area of the rectangle.

Problem: Find the dimensions of the rectangle of maximum area that can be inscribed in a right triangle with sides of length 3 and 4 and hypotenuse of length 5, if two sides of the rectangle lie along the two sides of the triangle. Make sure you justify that your answer is a maximum.

Solution:
Imagine the right triangle drawn with the x-axis and y-axis forming the two perpendicular sides and the line $y = -\frac{3}{4}x + 3$ forming the hypotenuse.
The area of the rectangle of dimensions a, b inscribed in the triangle is $A = ab$. How does this step help?
Then $f'(a) = -\frac{3}{4}a + 4$. (The critical number of f is $a = \frac{16}{7}$.) Why?
We check that $f''(a) < 0$ for $0 < a < \frac{16}{7}$ and $f''(a) > 0$ for $\frac{16}{7} < a < 4$. Thus $f(a)$ is maximized at $a = \frac{16}{7}$.
That the dimensions of the largest rectangle inscribed in such triangle are $a = \frac{16}{7}$ and $b = \frac{12}{7}$, show it! (Use the 1st or 2nd derivative test.)
- Find the length of the shortest ladder that can extend from a vertical wall, over a fence 2m high located 1m away from the wall, to a point on the ground beyond the fence.
- A cylindrical can is to be made to hold 1 liter of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.
- Show that of all the isosceles triangles with a given perimeter, the one with the greatest area is equilateral.
- A boat on the ocean is 4 km from the nearest point on a straight shoreline; that point is 6 km from a restaurant on the shore. A woman plans to row the boat straight to a point on the shore and then walk along the shore to the restaurant. If she walks at 3 km/hr and rows at 2 km/hr, at which point on the shore should she land to minimize the total travel time? If she walks at 3 km/hr, what is the minimum speed at which she must row so that the quickest way to the restaurant is to row directly (with no walking)?

Encouraging preliminary results

We used Mathematics Attitudes and Perceptions Survey (MAPS) to study the results. The basis for comparison was MAPS data collected in MATH 110 in April 2012. Assuming the population of Math 110 students does not significantly change through the years, we see some non-trivial improvements. Below we present the data for questions with the most significant improvement:



- Q2: After I study a topic in math and feel that I understand it, I have difficulty solving problems on the same topic.
- Q10: Understanding math means being able to recall something you’ve read or been shown.
- Q15: In math, it is important for me to make sense out of formulas and procedures before I use them.
- Q19: I often have difficulty organizing my thoughts during a math test.
- Q26: School mathematics has little to do with what I experience in the real world.
- Q30: If I get stuck on a math problem, there is no chance that I will figure it out on my own.

Overall MAPS average agreement with the experts:
27% for the 2015 group versus 11% for the 2012 group.

Student comments

“Explaining worked solutions was very useful in my understanding how to solve the problem as well as learning where I was going wrong. Having to explain solutions made sure that I had to be confident I knew why I did each step which was helpful.”

“I was doubtful on the usefulness at the start but going through the problems step-by-step with teacher and students helps.”

“Showing how problems/questions or approached step-by-step help. Being asked to explain how certain values are obtained, helped me understand how each step is related to another.”

Drawbacks

As with any technique, there are some drawbacks in its implementation.

- Time-consuming.
- Need a good instructor/TA to student ratio.
- Paper heavy.

